M.SC. FOURTH SEMESTER EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 42154**

Course Code: Math-404ME **Course Title: Computational Fluid Dynamics**

Full Marks: 40 **Time: 2 Hours**

The figures in the right hand side margin indicate full marks.

Notations and symbols have their usual meaning

Answer any five from the following questions. **8×5=40**

1. (a) Define various types of boundary conditions that are encountered in Computational Fluid Dynamics (CFD).

   (b) Define explicit and implicit schemes and give the example of these schemes. 4+4=8

2. (a) Prove that the function $u \in L^{\infty}_{loc}(\mathbb{R}_+ \times \mathbb{R})$ is a weak-solution of $u_t + f(u)_x$=0 with $u(x,0) = u_0(x)\ \forall\, x \in\ \mathbb{R}$ if the equation

   $\iint_{\mathbb{R}\mathbb{R}_+}(u\varphi_t + f(u).\,\varphi_x)dtdx + \int_{\mathbb{R}} u_0(x)\varphi(x,0)dx = 0$ is fulfilled for all test functions $\varphi \in C_0^1(\mathbb{R}_+ \times \mathbb{R})$.

   (b) Define the Rankine-Hugoniot condition for a discontinuous solution.

   (c) Define the entropy condition related to a discontinuous solution of the Cauchy problem.

   (d) Give an example of a Cauchy problem for scalar conservation law. 3+2+2+1=8

3. Derive Lax-Wendroff finite difference scheme for solving first order wave equation$u_t + cu_x$=0, $c > 0$ and hence discuss its stability analysis. 4+4=8

4. Use the FTCS Method to calculate a numerical solution of the equation $u_t = u_{xx}$,$0 < x < 1$, $t > 0$, where $(1)\ u = 0,\ \ x = 0$ and $1, t\ \geq 0, (ii)\ u = 2x\ , 0 \leq x \leq \frac{1}{2}\ , t = 0, (iii)\ u = 2(1-x),\ \frac{1}{2} \leq x \leq 1\ ,\ t = 0\ \left(\text{Take } \Delta x = \frac{1}{10},\ \Delta y = \frac{1}{100}\right)$. 8

5. Give an elaborate account of solving Navier-Stokes equations for incompressible two-dimension flows in cartesian coordinates using the MAC method. 8

6. (a) What is alternating direction implicit (ADI) technique? Explain.

(b) Write down explicit upwind differencing scheme and implicit upwind differencing scheme for the Linear Advection Equation $u_t + au_x$=0, $a > 0$. 4+4=8

7. Solve the following two-dimensional elliptic model mixed BVP;

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; \;\; 0 \le x \le 1, 0 \le y \le 1,$$

subject to the conditions:

$u(x, 0) = 2x, \qquad u(x, 1) = 2x - 1; 0 \le x \le 1,$

$u(0, y) + \frac{\partial u}{\partial x}(0, y) = 2 - y, \;\; u(1, y) = 2 - y; \;\; 0 \le y \le 1.$

Use the five-point formula with $h = \frac{1}{3}$ and $k = \frac{1}{3}$. 5+3=8

8. (a) What is Computational Fluid Dynamics?

(b) Why do we need computational methods for fluid dynamics problems?

(c) Define incompressible flow and irrotational flow.

(d) Give two examples of real-life problems involving incompressible flow. 2+2+2+2=8

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